

~: FREQUENCY RESPONSE ANALYSIS :~

CORRELATION BETWEEN TIME & FREQUENCY

RESPONSE:

- In time domain analysis of various i/p signal like step signal, ramp signal & parabolic signal are used for study the perform of control system.
- The time domain Specification of 2nd order system are delay time, rise time, peak time, maxth, over-shoot, settling time.
- Freq. response of a system is the response of the system when sinusoidal i/p signal is given. In gene - really the sinusoidal i/p signal is given as →

$$[r(t) = A \sin \omega t]$$

Whence, A = magnitude

ω = freq. of oscillation

- So, for this sinusoidal i/p the system gives a sinusoidal steady state o/p

$$\text{Suppose, } [c(t) = B \sin(\omega t - \phi)]$$

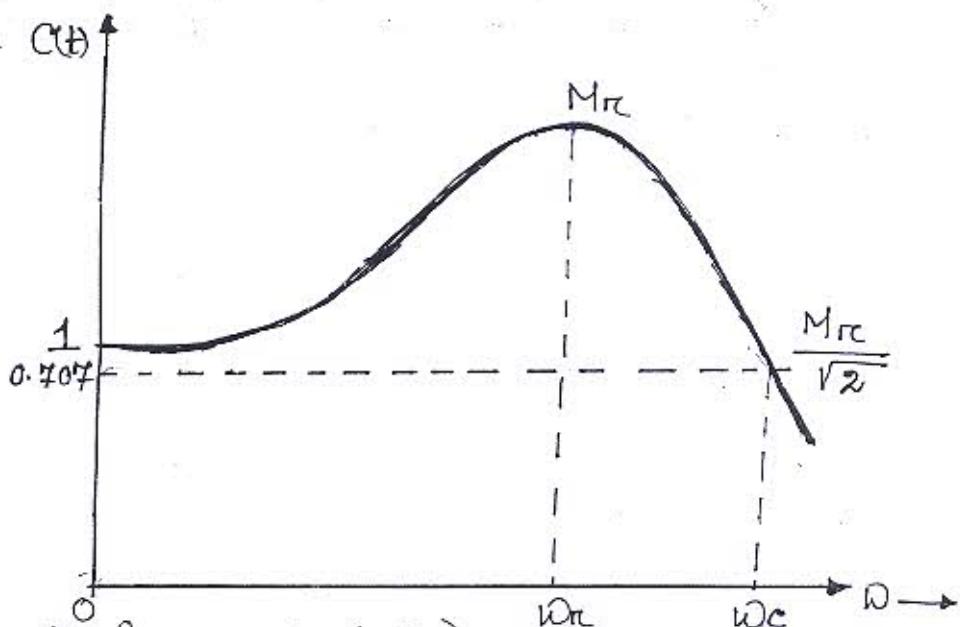
Whence, B = change in magnitude.

ϕ = phase angle betw i/p & o/p signal

ω = freq. of oscillation.

- The relation between the sinusoidal i/p & steady state o/p of a system is known as freq. response analysis.

FREQUENCY DOMAIN SPECIFICATION



- ① Resonant frequency (ω_r)
- ② Resonant peak (M_r)
- ③ cut off frequency (ω_c)
- ④ Band width (ω_B)
- ⑤ cut off rate
- ⑥ Gain margin
- ⑦ phase margin

① RESONANT FREQUENCY (ω_r):

→ It is the frequency at which the system has max^m magnitude.

② RESONANT PEAK (M_r):

→ Max^m value of magnitude is called resonant peak.

③ CUTOFF FREQUENCY (ω_c):

→ It is the freq. at which magnitude is $\frac{1}{\sqrt{2}}$ times less than the max^m value.

④ BANDWIDTH (ω_B):

→ It is the range of freq. over which the magnitude is equal to the $\frac{1}{\sqrt{2}}$.

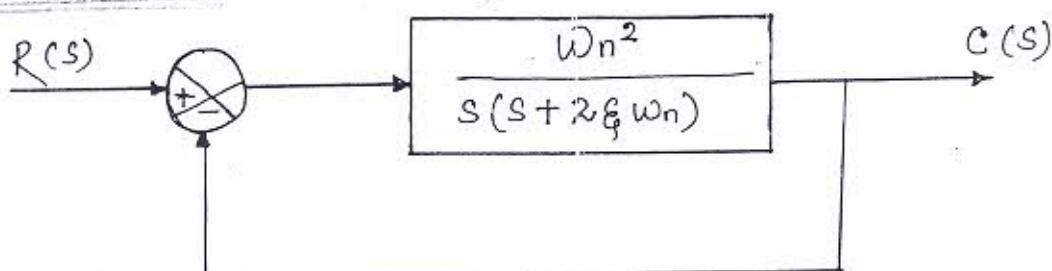
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⑤ CUTOFF RATE:

→ It is the rate of change of slope of magnitude at cut off frequency.

FREQUENCY RESPONSE BEHAVIOR OF 2ND ORDER SYSTEM:

SYSTEM :



MAGNITUDE :

$$M = \frac{1}{\sqrt{(1-\omega^2)^2 + 4\xi^2\omega^2}}$$

Where,

$$\omega = \frac{\omega_n}{\xi}$$

$$\text{phase angle } \phi = -\tan^{-1} \left(\frac{2\xi\omega}{1-\omega^2} \right)$$

$$\text{Resonant freqn, } \omega_r = \omega_n \sqrt{1-2\xi^2}$$

$$\text{Resonant peak, } M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\text{Bandwidth, } \omega_B = \omega_n \sqrt{(1-2\xi^2) + \sqrt{4\xi^4 - 4\xi^2 + 2}}$$

Q1 Determine the resonant freqn (ω_r), resonant peak (M_r) & bandwidth (ω_B) for

$$\frac{C(s)}{R(s)} = \frac{5}{s^2 + 2s + 5}$$

Sol:
C/S eqn, $1 + G(s)H(s) = 0$
 $\Rightarrow s^2 + 2s + 5 = 0$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

So, the C/S eqn of 2nd order system, $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

Then, $2\xi\omega_n = 2$ & $\omega_n^2 = 5$

$$\Rightarrow \xi = \frac{2}{2 \times 2.23}$$

$$\Rightarrow \omega_n = \sqrt{5}$$

$$\Rightarrow \omega_n = 2.23$$

$$\Rightarrow \xi = 0.44$$

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✓ Resonant freqⁿ, $\omega_{rc} = \omega_n \sqrt{1-2\zeta^2}$

$$= 2.23 \sqrt{1-2(0.44)^2}$$

$$\Rightarrow \boxed{\omega_{rc} = 1.745}$$

Resonant peak, $M_{rc} = \frac{1}{2\zeta \sqrt{1-\zeta^2}}$

$$= \frac{1}{2 \times 0.44 \sqrt{1-(0.44)^2}}$$

$$\Rightarrow \boxed{M_{rc} = 1.265}$$

Bandwidth, $\omega_B = \omega_n \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$

$$= 2.23 \sqrt{(1-2(0.44)^2) + \sqrt{4(0.44)^4 - 4(0.44)^2 + 2}}$$

$$\Rightarrow \boxed{\omega_B = 2.97}$$

POLAR PLOT:

- ① It is the plot of the T.F.G(s) in the polar co-ordinate when w varies from 0 to ∞ . It is the locus of the magnitude versus phase angle of $G(jw)$, where w varies from 0 to ∞ .

- ② This plot is used to determine the stability of open loop T.F. i.e. $G(s)$ in the help of gain margin & phase margin.

PROCEDURE TO DRAW POLAR PLOT:

STEP: 01

- Det. the open loop T.F. putting $s = jw \Rightarrow G(s) = G(jw)$.

STEP: 02

- Det. the magnitude of $|G(jw)|$ and phase angle of i.e. $\angle G(jw)$ where w varies from 0 to ∞

STEP: 03

- Rationalize the T.F. $G(jw)$ and equate the real part is equal to zero & imaginary part is equal to zero.

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- a) if Real part is equal to zero, then the freqⁿ. at that point, determine the magnitude and corresponding phase angle. At that time the plot cut the Imaginary axis.
- b) if imaginary part = 0, at that freqⁿ, determine magnitude and corresponding phase angle. Hence at that time the plot cut the real axis.

STEP:4

→ at various values of ω i.e from 0 to ∞ , draw the locus of magnitude versus phase angle plot. This plot is known as polar plot.

Q.2 Sketch the polar plot for the given T.F, $G(s) = \frac{20}{s(s+1)(s+2)}$

step 1

$$\text{Put } s = j\omega$$

$$G(s) = \frac{20}{s(s+1)(s+2)}$$

$$\Rightarrow G(j\omega) = \frac{20}{j\omega(j\omega+1)(j\omega+2)}$$

step 2

$$|G(j\omega)| = \frac{20}{\sqrt{\omega^2} \cdot \sqrt{\omega^2 + 1} \cdot \sqrt{\omega^2 + 4}}$$

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{2}\right) = \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

\therefore (If ω is present in the denominator then took (\tan^{-1})).

$$\therefore \tan^{-1}\left(\frac{\omega}{0}\right) = \tan^{-1}\infty = \frac{\pi}{2}$$

$$\text{So, } \lim_{\omega \rightarrow 0} \angle G(j\omega) = -\frac{\pi}{2} \Rightarrow \phi = -\frac{\pi}{2}$$

$$\lim_{\omega \rightarrow 0} |G(j\omega)| = M = \infty$$

$$\text{Again, } \lim_{\omega \rightarrow \infty} |G(j\omega)| = M = \frac{20}{\infty} = 0$$

$$\lim_{\omega \rightarrow \infty} \angle G(j\omega) = \phi = -\frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} = -\frac{3\pi}{2}$$

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Step 3Rationalize :

$$G(j\omega) = \frac{20(-j\omega)(-j\omega+1)(-j\omega+2)}{(j\omega)(-j\omega)(j\omega+1)(-j\omega+1)(j\omega+2)(-j\omega+2)}$$

$$= \frac{-60\omega^2}{\omega^2(1+\omega^2)(2+\omega^2)} + j \frac{20(\omega^3 - 2\omega)}{\omega^2(1+\omega^2)(2+\omega^2)}$$

Equating imaginary part = 0

$$\Rightarrow \frac{20(\omega^3 - 2\omega)}{\omega^2(1+\omega^2)(2+\omega^2)} = 0$$

$$\Rightarrow 20(\omega^3 - 2\omega) = 0$$

$$\Rightarrow \omega^3 - 2\omega = 0$$

$$\Rightarrow \omega^3 = 2\omega$$

$$\Rightarrow \omega^2 = 2$$

$$\Rightarrow \omega = \sqrt{2}$$

∴ put $\boxed{\omega = \sqrt{2}}$

$$M = |G(j\omega)| = \frac{20}{\sqrt{2} \cdot \sqrt{2+1} \cdot \sqrt{2+4}} = \frac{20}{\sqrt{2} \cdot \sqrt{3} \cdot \sqrt{6}} \\ = \frac{20}{\sqrt{36}}$$

$$\Rightarrow \boxed{M = \frac{10}{3}}$$

$$\phi = \angle G(j\omega) = -\tan^{-1}\left(\frac{\sqrt{2}}{2}\right) - \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) - \tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\pi}{2} - 54.43 - 35.26$$

$$= -\frac{\pi}{2} - 89.99$$

$$= -\frac{\pi}{2} - 90^\circ (\because 89.99 \approx 90)$$

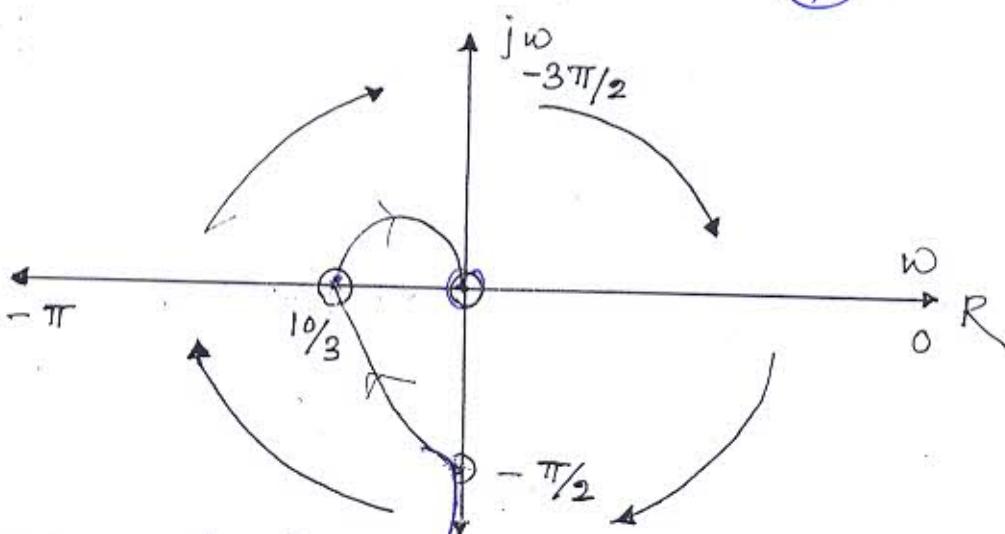
$$\Rightarrow \boxed{\phi = -\pi}$$

$$\therefore \boxed{\omega = 0, M = \infty, \phi = -\frac{\pi}{2}}$$

$$\boxed{\omega = \sqrt{2}, M = 10/3, \phi = -\pi}$$

$$\boxed{\omega = \infty, M = 0, \phi = -3\frac{\pi}{2}}$$

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Q.3 Sketch the polar plot of the TF, $G(s) = \frac{1}{1+sT}$

el: put $s = j\omega$

$$G(j\omega) = \frac{1}{1 + j\omega T}$$

$$M = |G(j\omega)| = \frac{1}{\sqrt{1 + \omega^2 T^2}}$$

$$\phi = \angle G(j\omega) = -\tan^{-1}(\frac{\omega T}{1}) = -\tan^{-1}(\omega T)$$

$$\lim_{\omega \rightarrow 0} |G(j\omega)| = \frac{1}{\sqrt{1}} = 1$$

$$\lim_{\omega \rightarrow 0} \angle G(j\omega) = 0$$

$$\text{Again, } \lim_{\omega \rightarrow \infty} |G(j\omega)| = \frac{1}{\sqrt{1+\infty}} = 0$$

$$\lim_{\omega \rightarrow \infty} \angle G(j\omega) = -\tan^{-1}(\infty) = -\frac{\pi}{2}$$

By rationalize, we get,

$$\therefore \omega = \frac{1}{T}$$

$$\text{Putting } \omega = \frac{1}{T},$$

$$M = |G(j\omega)| = \frac{1}{\sqrt{1 + \frac{1}{T^2} T^2}} = \frac{1}{\sqrt{2}}$$

$$\phi = \angle G(j\omega) = -\tan^{-1}(1) = -\frac{\pi}{4}$$

$$\therefore \omega = 0, M = 1, \phi = 0$$

$$\omega = \frac{1}{T}, M = \frac{1}{\sqrt{2}}, \phi = -\frac{\pi}{4}$$

$$\omega = \infty, M = 0, \phi = -\frac{\pi}{2}$$

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closed loop stable system, because the closed loop stability depends on the roots of the $G(s)H(s)$ but not on the open loop system.

NYQUIST CONTOUR:

→ Consider a contour which encircles the full right half of the s-plane. Such a contour encircles all the right half s-plane zeros and poles of $G(s)H(s)$.

Path-1

$$\text{put } S = j\omega$$

where, ω varies from 0 to ∞ .

Path-2

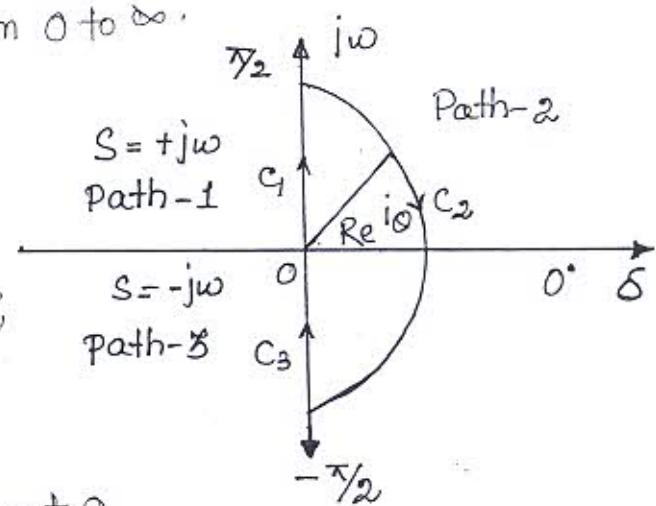
$$\text{put } S = \lim_{R \rightarrow \infty} Re^{j\theta}$$

Where, θ varies from $+\frac{\pi}{2}$ to $-\frac{\pi}{2}$ through 0;

Path-3

$$\text{put } S = -j\omega$$

where, ω varies from $-\infty$ to 0



STATEMENT OF NYQUIST CRITERIA:

→ If the contour, T_{GH} of the open loop transfer function $G(s)H(s)$ corresponding to the Nyquist contour in the s-plane encircles the point $(-1 + j0)$ in the counter clockwise direction as many times as the no. of right half s-plane poles of $G(s)H(s)$ the closed loop system is stable. In case of open loop stable system, the closed loop system is stable if contour, T_{GH} of $G(s)H(s)$ doesn't encircle $(-1 + j0)$ point that is the net encirclement is zero.
 $\because N = \text{no of encirclements by the contour } T_{GH} \text{ around the point } (-1 + j0)$

RELATIVE STABILITY ANALYSIS:

→ Gain Margin & phase Margin are the 2 quantities which defines the relative stability of a system.
 → The gain margin & phase margin are the 2 major of the stability in case of open loop stable system.

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GAIN MARGIN :

→ It is the factor by which the system gain can be increased to drive the system to range of instability.

$$\rightarrow \text{Gain Margin (G.M)} = \frac{1}{\alpha} = \frac{1}{|GH(j\omega)|}_{\omega=\omega_c}$$

→ In decimal unit, gain margin is,

$$[GM \text{ in dB} = 20 \log \frac{1}{\alpha}]$$

$\omega_c \rightarrow$ corner or cut-off frequency

∴ Since, for stable system, $\alpha < 1$, the gain margin for a stable system must be +ve.

∴ whence, ω_c is phase crossover frequency.

PHASE MARGIN:

→ It is defined as the amount of additional phase lag at the gain cross-over freqn (ω_g) required to bring the system to the range of instability.

$$[\therefore \Phi_{PM} = 180^\circ + \angle GH(j\omega) \Big|_{\omega=\omega_g}]$$

Whence, ω_g = gain cross-over freqn, that is defined as the frequency at which the gain is unity.

→ The PM is measured in the counter clockwise direction from the -ve real axis and it is always +ve for the stable system.

→ A large gain margin & phase margin indicates a very stable system.

Ex-1

A unit feedback system has a open loop transfer function

$$G(s) = \frac{(s+2)}{(s+1)(s+3)}$$

using Nyquist criteria. Let whether

the close loop system is stable/not.

do Path-1

$$\text{Put } s = j\omega$$

$$G(j\omega) = \frac{(j\omega+2)}{(j\omega+1)(j\omega+3)}$$

~: BODE PLOT:~

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→ It consist of 2 plots :-

(1) Magnitude of $|G(j\omega)H(j\omega)|$ in dB vs $\log_{10}\omega$

(2) phase angle of $\angle G(j\omega)H(j\omega)$ vs $\log_{10}\omega$

→ So we draw both the plot in logarithmic freq^o
hence we consider each term individually present in $G(s) \cdot H(s)$

→ This plot is used to determine to open loop stability accurately.

$$\text{In polar plot, } G(s) = \frac{K}{s(s+1)} = \frac{K}{j\omega(j\omega+1)}$$

$$\text{Magnitude, } M = \frac{K}{\omega\sqrt{\omega^2+1}}$$

$$\text{phase angle, } \phi = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{1}\right)$$

CORNER FREQUENCY:

→ It is the reciprocal of the co-efficient "S" in terms of $(1+as)$.

$$[w_c = \frac{1}{a}]$$

Example: $(b+s)$

corner freqⁿ. will be.

$$1 + \frac{s}{b}$$

coefficient of s is $\frac{1}{b}$

∴ So, corner freqⁿ is reciprocal of $\frac{1}{b} = b$.

Example:-

$$G(s) = \frac{K(s+2)}{s(s+3)(1+5s)}$$

$$(s+2) \rightarrow 1 + \frac{s}{2},$$

$$\frac{1}{2} = 2$$

$$(s+3) \rightarrow 3$$

$$(1+5s) \rightarrow \frac{1}{5} = 0.2$$

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∴ $0.2, 2, 3$

$$\therefore G(s) = \frac{K \times 2 (s+2)}{s(s+3) \times 3(1+5s)}$$

$$= \frac{2K}{3}, \frac{1}{s}, 0.2, 2, 3$$

Factors	ω_c	Slope in dB/decade	Net slope
$\frac{2K}{3}$	—	0	0
$\frac{1}{s}$	—	-20dB/decade	-20dB/decade
$\frac{1}{1+5s}$	0.2	-20dB/decade	-40 dB/decade
$1 + \frac{s}{2}$	2	20dB/decade	-20 dB/decade
$1 + \frac{s}{3}$	3	-20dB/decade	-40 dB/decade

→ When one zero value is present, take +20 dB/decade
 Similarly, for one plot, take -20 dB/decade.

Q What do you mean by decade?

A Decade means if the freq? is increased 10 times than
 the magnitude decreases -20dB. So slope is -20 dB/decade
 Let, $\omega = 1$, it increases to $\omega = 10$

Then, $M = -20 \text{ dB}$.

Q What do you mean by octave?

A It is the freq? increase 2 times than magnitude
 decreases -6dB.

Hence, the slope is -6 dB/octave .

Let, $\omega = 1$, it increases, $\omega = 2$

Slope = -6 dB/octave

DIFFERENT TYPE OF THE SYSTEM

TYPE-0

$$G(s) \cdot H(s) = \frac{K}{(s)^0}$$

$$G(j\omega) \cdot H(j\omega) = \frac{K}{(j\omega)^0}$$

$$\text{Magnitude } (M) = 20 \log K - 20 \log (j\omega)^0$$

$$\Rightarrow [M = 20 \log K]$$

Initial slope = 0 dB/decade upto the first w.

$$\Rightarrow [\omega = K] \quad [\because K = \text{gain}]$$

TYPE-1

(One pole is present at origin)

$$G(s) \cdot H(s) = \frac{K}{s^1}$$

$$G(j\omega) \cdot H(j\omega) = \frac{K}{(j\omega)^1}$$

$$\text{Magnitude } (M) \text{ in dB} = 20 \log K - 20 \log j\omega$$

$$= 20 \log K - 20 \log \omega$$

When it reaches to 0 $\Rightarrow [K = \omega]$

Taking initial slope = -20 dB/decade.

TYPE-2

$$G(s) \cdot H(s) = \frac{K}{s^2}$$

$$G(j\omega) \cdot H(j\omega) = \frac{K}{(j\omega)^2}$$

$$M = 20 \log K - 20 \log |j\omega^2|$$

$$M = 20 \log K - 20 \log \omega^2$$

When reaches to 0, $[\omega = \sqrt{K}]$

Initial slope = -40 dB/decade

Starting point = $20 \log \frac{K}{\omega}$

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After starting point slope reduces to -40 dB/decade
upto $w = \sqrt{K}$

EXAMPLE:

$$(1) G(s)H(s) = \frac{31.63}{1+1.25s} \quad \text{Draw the Bode plot.} \quad a = 1.25$$

Ans: corner freqⁿ (ω_c) = $\frac{1}{1.25} = 0.8 \text{ rad/sec}$

So, the freqⁿ range is $\omega = 0.1$ to $\omega = 1.0$
(it depends on ω_c)

It is a type-0 system, so initial slope = 0 dB/decade.

$$\text{Starting point} = 20 \log K = 20 \log (31.63) \\ = 30^\circ \text{ dB at } \omega = 0.1$$

$$30^\circ \text{ dB continues till, } \omega = K = 31.63$$

$$\text{Net slope} = 0 - 20 = -20 \text{ dB/decade}$$

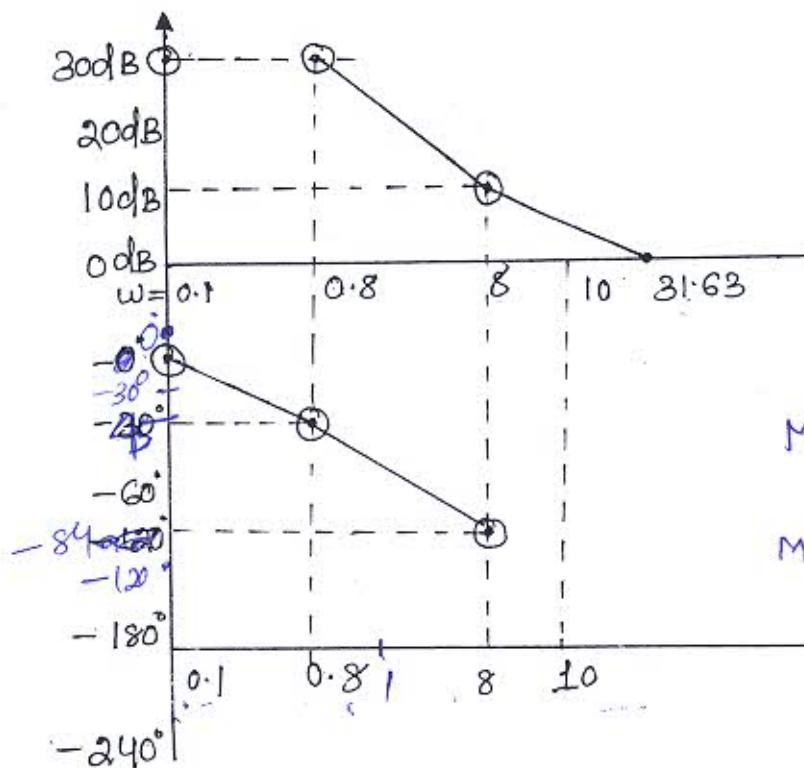
$$H(j\omega) = \frac{31.63}{1+1.25(j\omega)}$$

$$\phi = \angle G(j\omega) H(j\omega) = -\tan^{-1} \left(\frac{1.25\omega}{1} \right)$$

$$\omega = 0.1, \phi = -7.125^\circ$$

$$\omega = 0.8, \phi = -45^\circ$$

$$\omega = 8, \phi = -84^\circ$$



$$M = \frac{K}{\omega \sqrt{\omega^2 + 1}}$$

$$M = \frac{31.63}{(0.1)\sqrt{(0.1)^2 + 1}}, \omega = 0.1$$

$$M = 31.31.$$

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$$M = \frac{K}{\omega \sqrt{\omega^2 + 1}}$$

$$M = \frac{31.63}{(0.1) \sqrt{(0.1)^2 + 1}}, \omega = 0.1$$

$$M = 31.81$$

